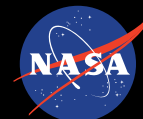




# Multilayer Clustered Sampling Technique (MLCS) for Near-Earth Asteroid Impact Hazard Assessment

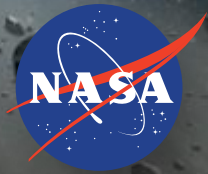
Javier Roa and Davide Farnocchia



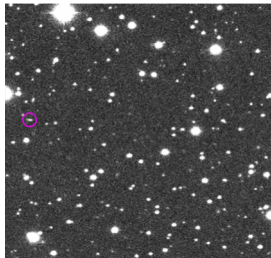
**Jet Propulsion Laboratory**  
California Institute of Technology

# Impact Hazard Assessment

## Introduction



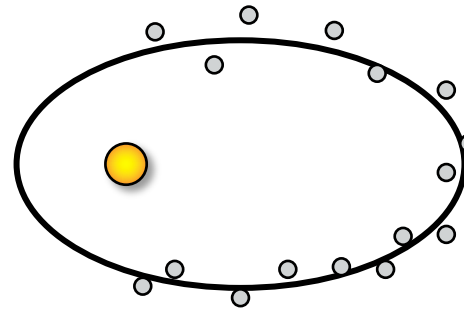
### OBSERVATIONS



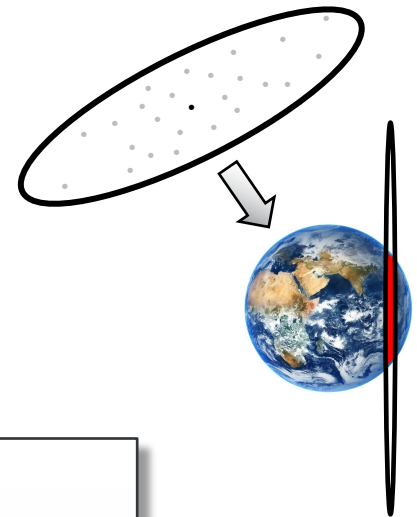
### ASTROMETRY

Date	R. A.	Dec.
2019-01-14.53	23.5	30.0
2019-01-14.54	23.4	30.1
2019-01-14.55	23.3	30.2

### ORBIT FIT



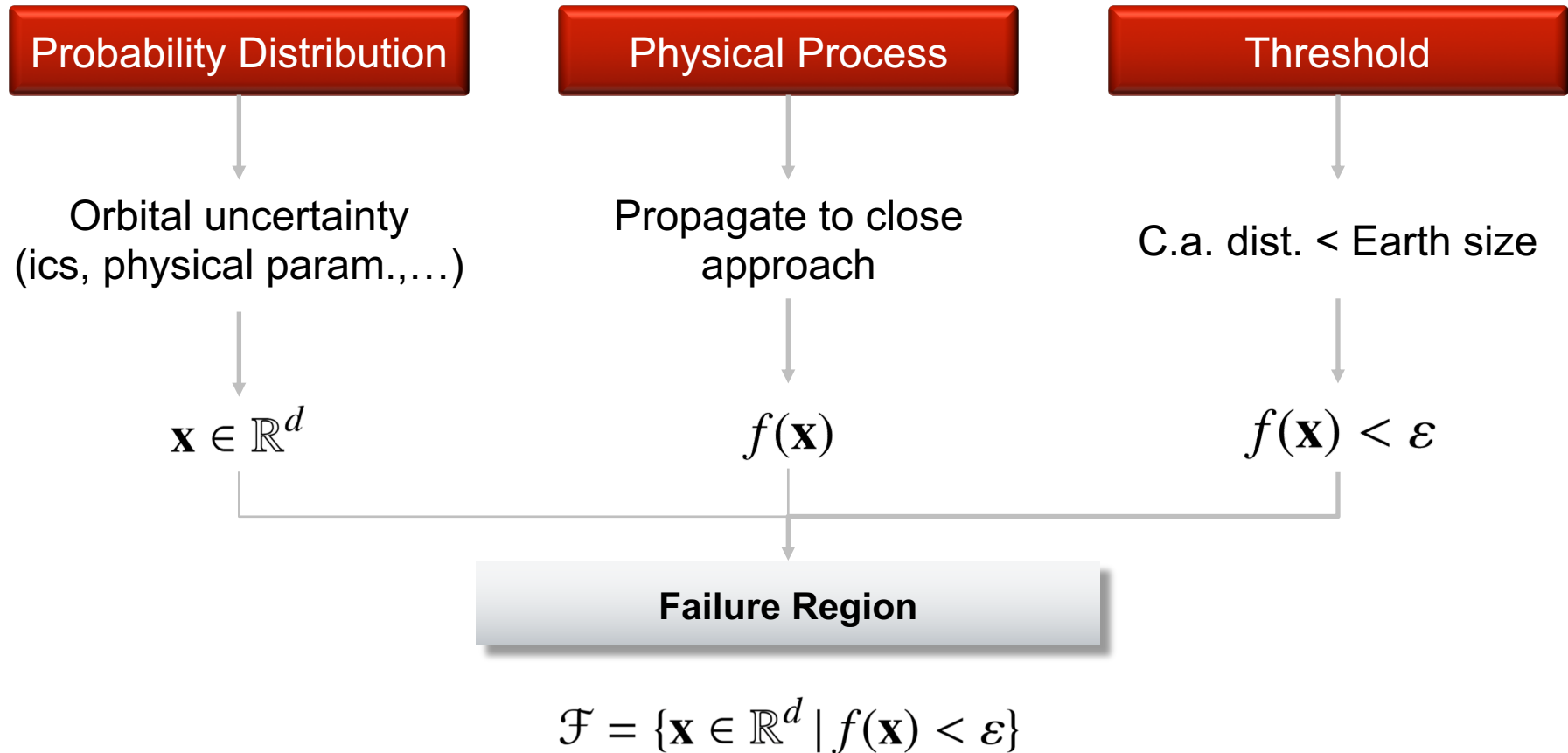
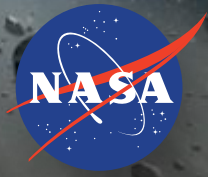
### HAZARD ASSESS.



- Small probability ( $\sim 10^{-7}$ ).
- Distribution not necessarily Gaussian.
- Planetary encounters  $\rightarrow$  Strongly nonlinear.
- To be implemented into an automatic system (Sentry).
- No human interaction.

# Rare Event Detection

Generic problem



The failure region needs not be connected!

# MLCS

## Multi-Layer Clustered Sampling



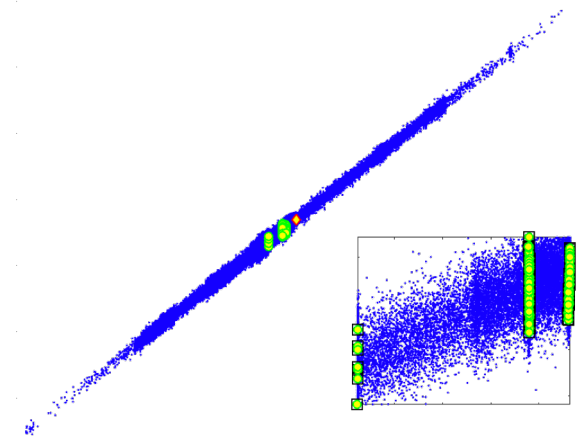
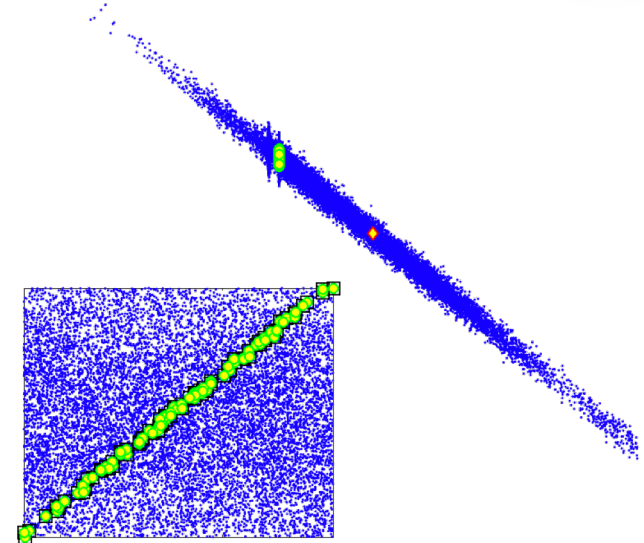
### Locating $\mathcal{F}$

$$P = \frac{N_{\text{impacts}}}{N_{\text{samples}}}$$

If  $P$  is small  $\Rightarrow \mathcal{F}$  is small.  
Locate a small subset of i.cs.

### MLCS

- As accurate as Monte Carlo sampling.
- The smaller  $P$ , the greater the speedup.
- No need for proposal distributions.





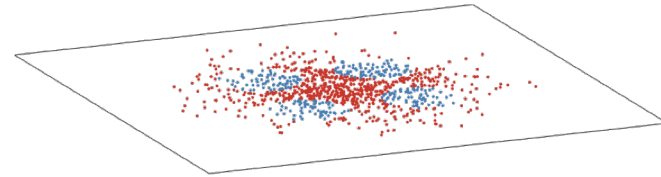
# MLCS II

## Multi-Layer Clustered Sampling

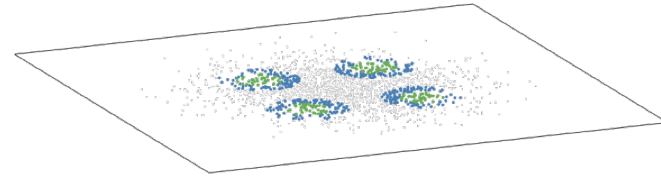


### MLCS Algorithm

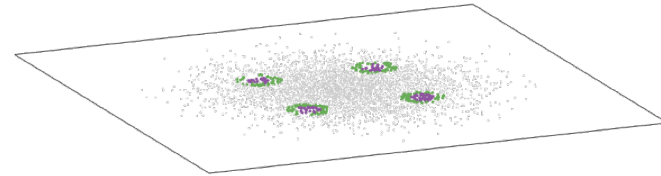
1. Sample layers
2. Evaluate  $f(\mathbf{x})$  on layer 1.
3. Check convergence.
4. Select top  $p$ -percentile.
5. Cluster points.
6. Advance to the next layer.
7. Repeat 3-6 until converged.



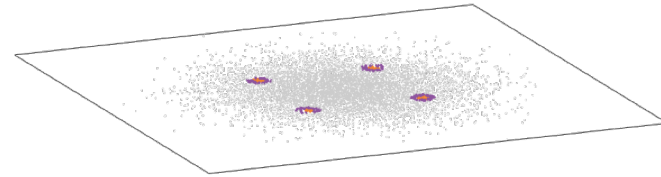
$$N_1$$



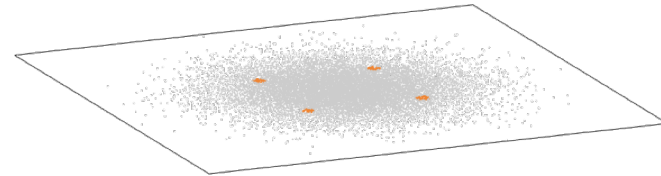
$$N_2 = \ell N_1$$



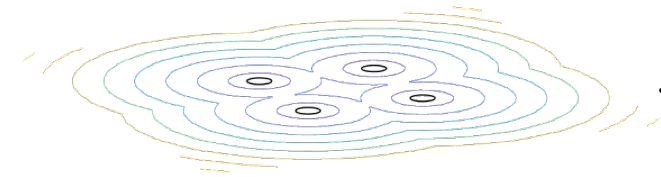
$$N_3 = \ell^2 N_1$$



$$N_4 = \ell^3 N_1$$



$$N_5 = \ell^4 N_1$$



$$f(\mathbf{x})$$

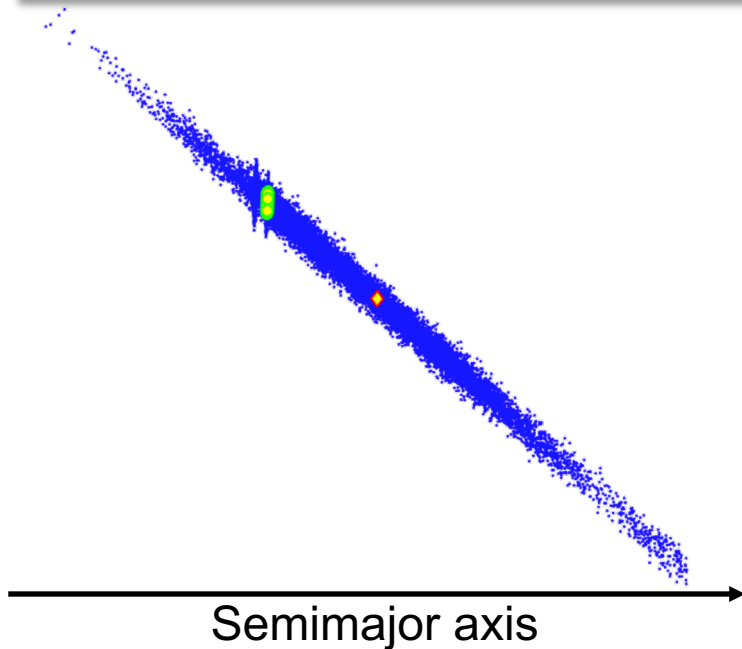
# Dynamical Regimes

Clustering in orbital-element space



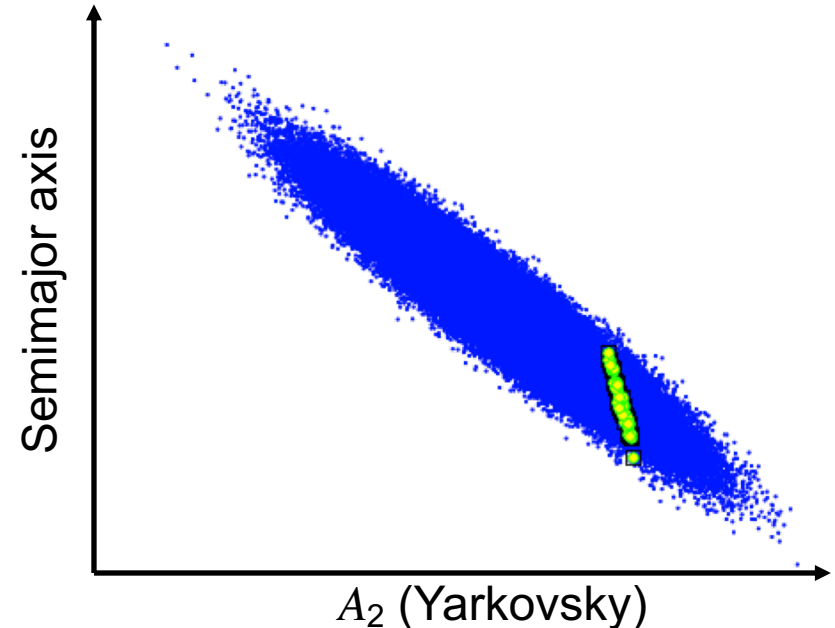
## Orbital Elements

The uncertainty of the semimajor axis tends to dominate the failure region



## Physical Parameters

The uncertainty of other physical parameters might play an even more important role

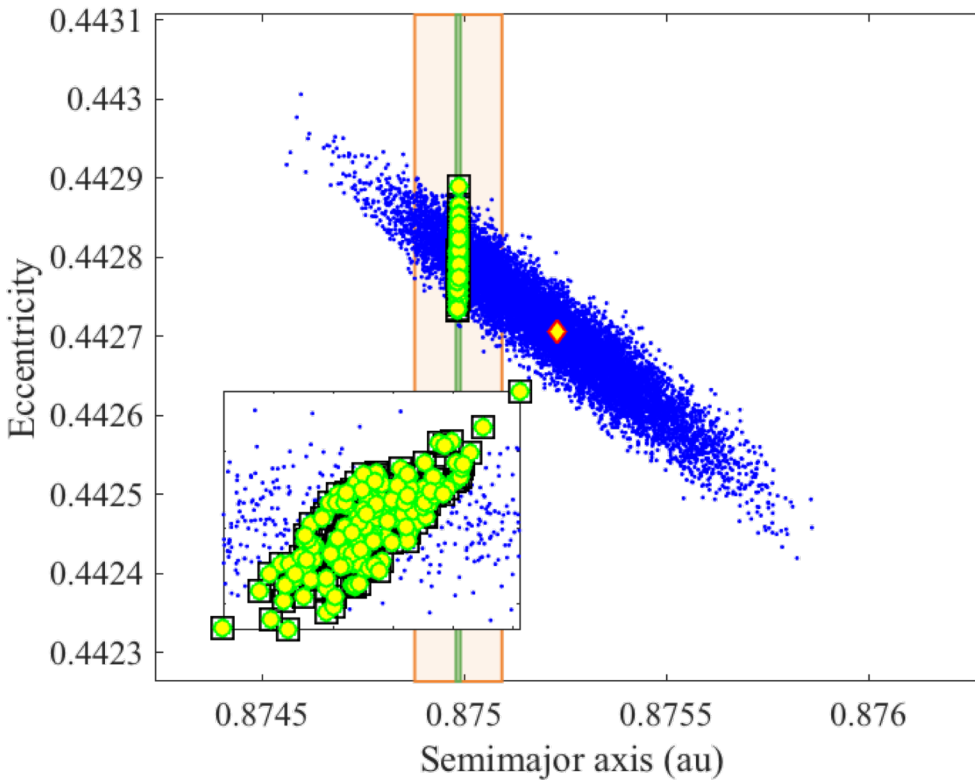


# Example I

## 2017 RH16

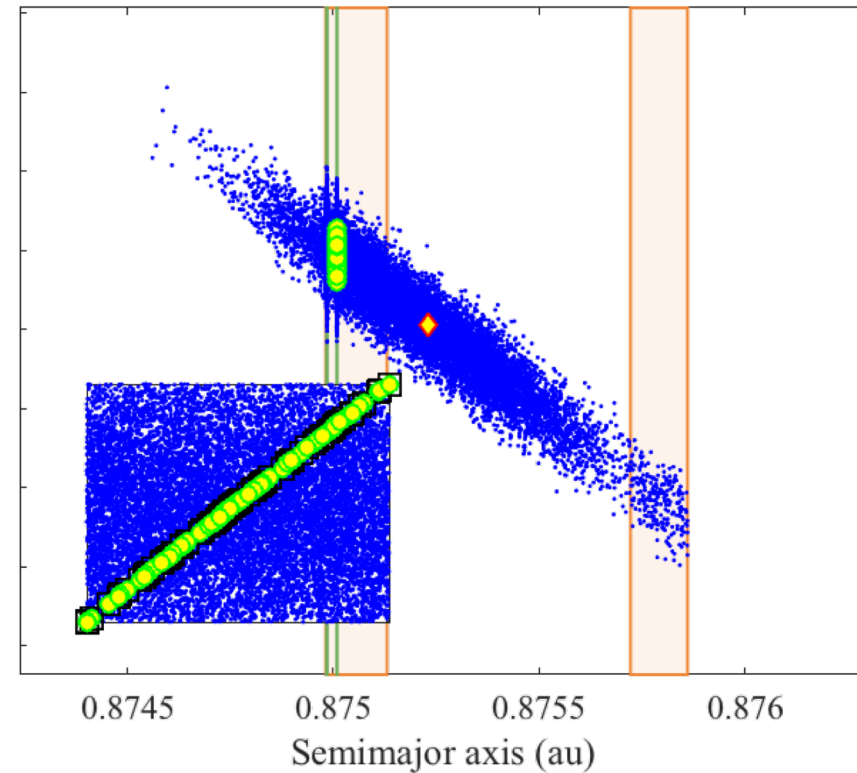


$P = 9.9 \times 10^{-4}$  (2026-Aug-31)



×10      Speedup (adaptive)  
×1211    Speedup (absolute)

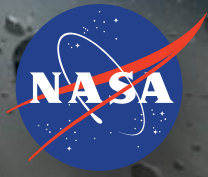
$P = 5.1 \times 10^{-6}$  (2031-Sep-01)



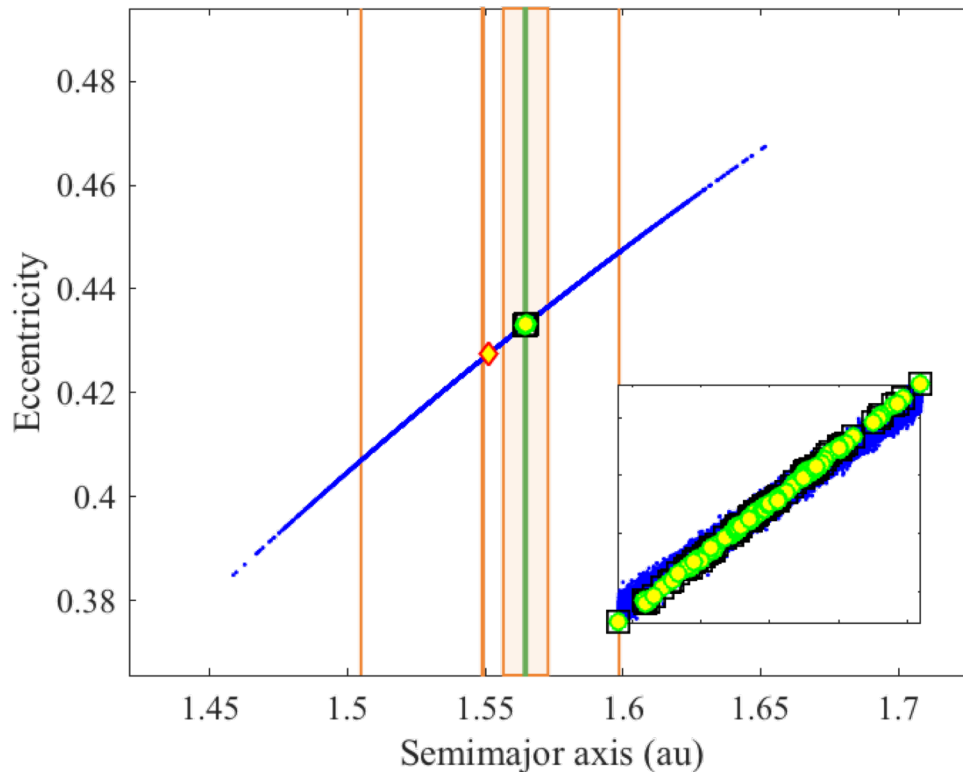
×276      Speedup (adaptive)  
×276      Speedup (absolute)

# Example II

## 2013 YB

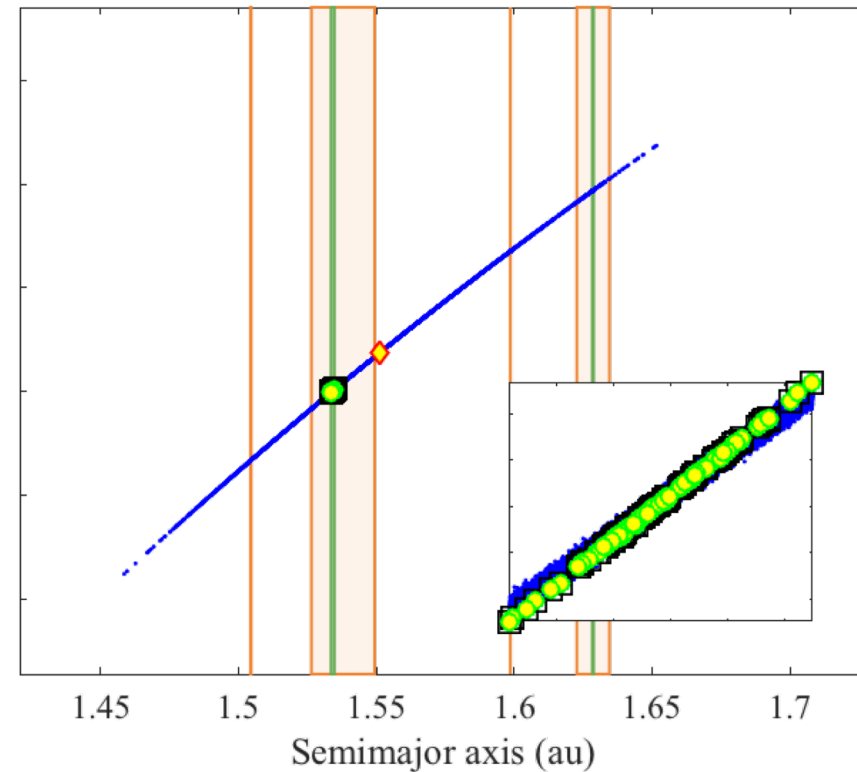


$P = 2.0 \times 10^{-5}$  (2023-Dec-24)



×33      Speedup (adaptive)  
×131     Speedup (absolute)

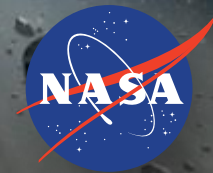
$P = 2.1 \times 10^{-5}$  (2024-Dec-23)



×35      Speedup (adaptive)  
×130     Speedup (absolute)

# Example III

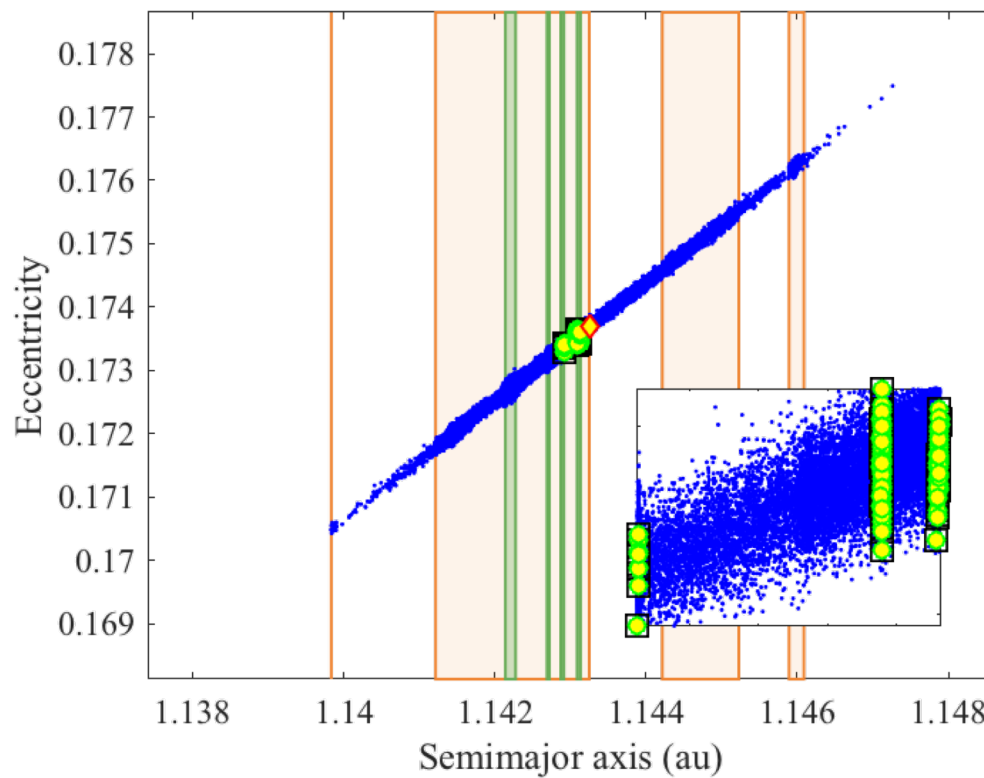
2018 UM1



$P = 1.4 \times 10^{-6}$  (2095-Jun-09.08)

$P = 1.8 \times 10^{-5}$  (2095-Jun-09.39)

$P = 1.6 \times 10^{-5}$  (2095-Jun-09.40)



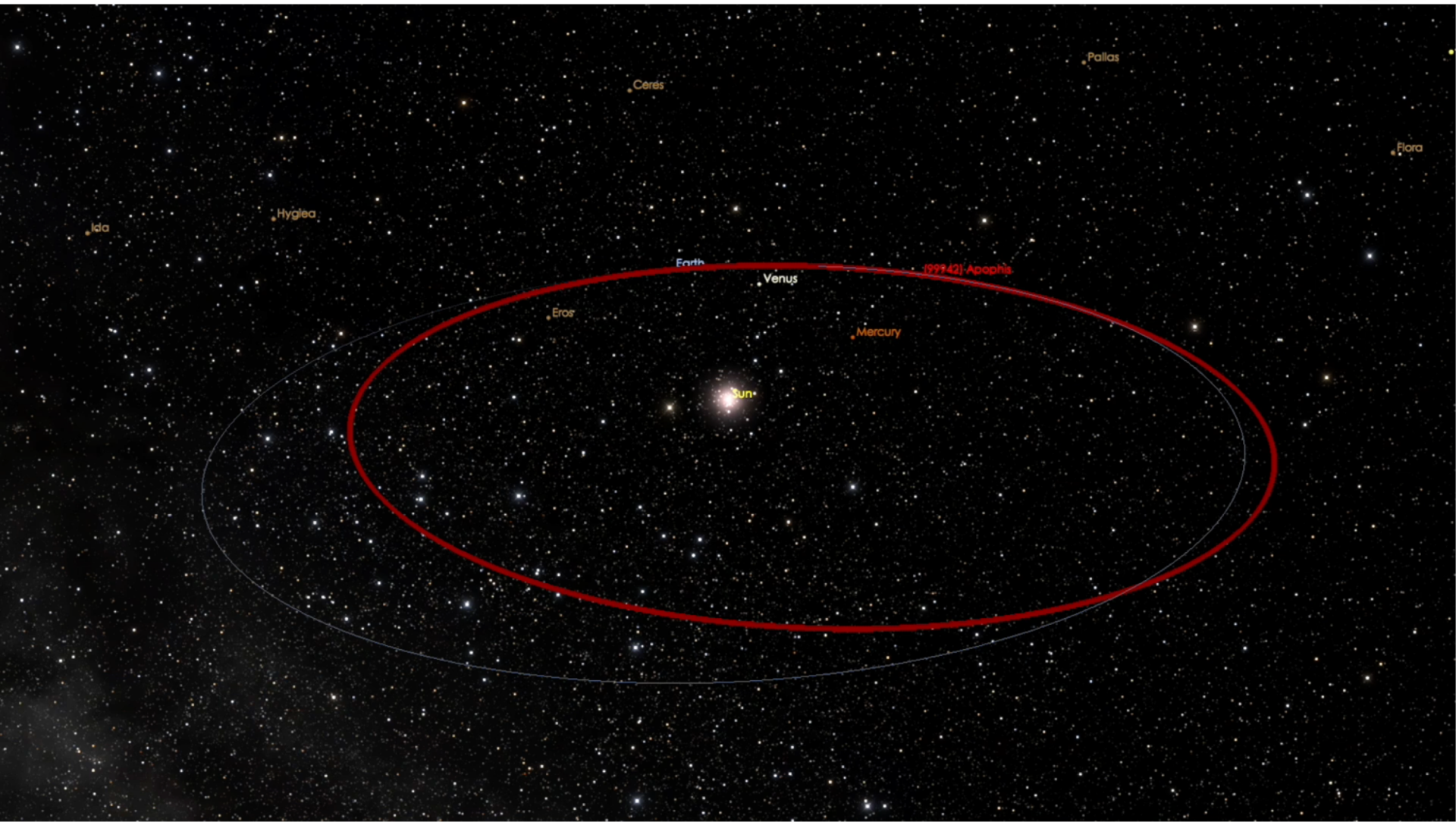
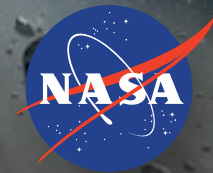
×15      Speedup (adaptive)

×61      Speedup (absolute)



# Example IV

(99942) Apophis



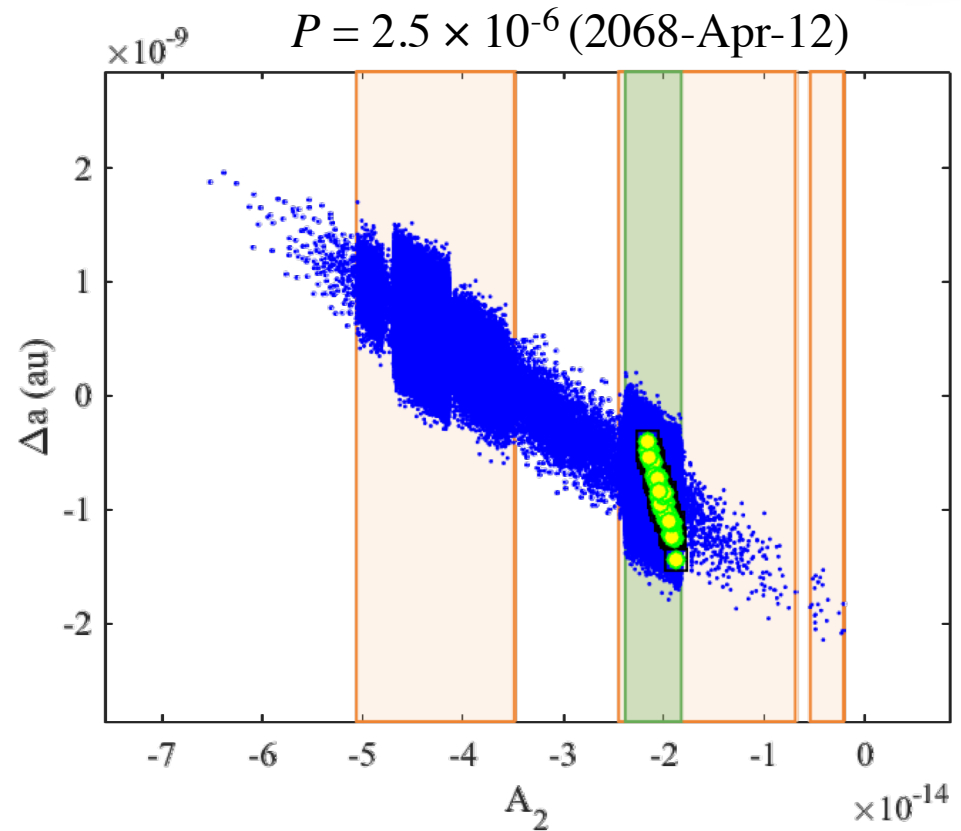
# Example IV

## (99942) Apophis



### 2068 Close Approach

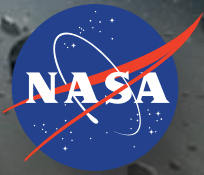
- Strongly nonlinear due to 2029 close approach.
- Driven by the Yarkovsky effect.
- Linearized methods fail.



×12 Speedup (adaptive)

×12 Speedup (absolute)

# Conclusions



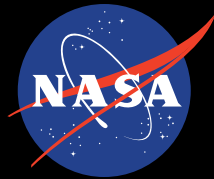
## MLCS as a generic algorithm

1. MLCS is an efficient alternative to Monte Carlo sampling:
  - Retains accuracy.
  - Significant speedups, especially for estimating low probabilities.
2. No assumptions about the uncertainty distribution.
3. No dynamical assumptions/simplifications.

## MLCS for impact monitoring

1. Fully nonlinear.
2. Handles any source of perturbation.
3. Separates individual Virtual Impactors thanks to clustering.
4. Requires no human interaction (adequate for automatic systems).
5. Handles uncertain physical parameters.

*“MLCS is an efficient alternative to  
Monte Carlo sampling”*



**Jet Propulsion Laboratory**  
California Institute of Technology

[javier.roa@jpl.nasa.gov](mailto:javier.roa@jpl.nasa.gov)